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BRST anomaly and superspace constraints of the pure spinor heterotic string in a curved background

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Abstract: The pure spinor heterotic string in a generic super Yang-Mills and supergravity background is considered. We determine the one-loop BRST anomaly at the cohomological level. We prove that it can be absorbed by consistent corrections of the classical constraints due to Berkovits and Howe, in agreement with the Green-Schwarz cancelation mechanism.

Keywords: BRST Symmetry, Superstrings and Heterotic Strings.

Contents

1. Introduction

The pure spinor formalism of superstrings [1] and the Green-Schwarz (GS) formalism [2] are synoptical, in that both are based on the embedding of the superstring world-sheet into the target superspace and have manifest supersymmetry. The deep relation between the two approaches is discussed in [3 − 5]. The pure spinor approach has the great advantage over the GS one to allow for a consistent and covariant quantization of the superstring. The GS approach cannot be quantized covariantly due to its peculiar κ-symmetry [6] which cannot be gauge fixed in a Lorentz covariant way. In the pure spinor formulation the κ-symmetry is replaced by a symmetry generated by the BRST charge $Q = \oint \lambda^\alpha d_\alpha$ where λ^α is a pure spinor.

One of the main applications of the pure spinor formalism is the construction of string actions on supersymmetric backgrounds [7, 8, 3], including those with Ramond-Ramond fields like anti de Sitter space-times [9]. A common feature of both approaches in curved backgrounds is that, in the associated σ-models, the requirement of invariance and nilpotence under κ-symmetry in one case [10] and under the BRST symmetry in the other case [7, 8], implies, to zero order in α', constraints for the background torsion and curvatures that force the background fields to be on shell.

A relevant question in this context is to understand and compute the corrections to these constraints to higher order in α'.

There are two ways to study these corrections for the GS superstring. The first method computes the relevant β-functions and imposes that they vanish to reach conformal invariance at the quantum level. The vanishing of the β-functions determines the corrections to the background field equations [11]. The second method is cohomological in nature: it classifies and then computes the anomalies of the BRST κ-symmetry. These anomalies determine the α' corrections of the torsion and curvatures constraints [12 − 14]. The equivalence of the two methods becomes clear if one notices that the square of the κ-symmetry

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transformations produces a Weyl-Lorentz (i.e. conformal) world-sheet transformation. It is remarkable that correction of order n in α' in the conformal approach appear at order $(n-1)$ in the cohomological approach.

Since the pure spinor formulation describes a critical string, one expects that the conformal invariance is preserved on shell also at the quantum level and the β-functions vanish [15, 16]. However, as discussed in [7], the vanishing of the β-function is not sufficient to determine the corrections of the field equations. It turns out that the holomorphicity of the BRST current and the nilpotence of the BRST charge are also needed. Equivalently one can apply the cohomological method and study the anomalies of the BRST symmetry generated by Q.

For the heterotic string, both in the GS and in the pure spinor approaches, the constraints that arise at zero order in α' describe a model where the B-field is decoupled from the gauge sector. Then, at first order in α', one expects a correction related to the gauge and Lorentz Chern-Simons three-form, in order to cancel the gauge and Lorentz anomalies by the standard Green-Schwarz mechanism [17].

In the GS formulation, this correction was indeed found, as an anomaly of the κ-symmetry, for the Yang-Mills Chern-Simons form in [12] and for the full (gauge and Lorentz) Chern-Simons form in [13]. The coefficients of this anomaly has been explicitly computed in [14], in agreement with the GS anomaly cancelation mechanism. One should notice that in order to implement the consistency condition for the Lorentz anomaly, a theorem to obtain a solutions of the SUGRA-SYM constraints in presence of the gauge and Lorentz Chern-Simons forms has to be used [18−20].[1]

In this paper we consider the problem of determining the α' corrections of the heterotic string σ-model, in the framework of the pure spinor approach, looking for the BRST anomalies at the cohomological level. In particular we shall obtain the full expression of the anomaly related to the gauge and Lorentz Chern-Simons three-form, which arises at first order in α'.

In the next section we will review the pure spinor construction for the heterotic string in a generic SYM/SUGRA background. In section 3 we determine the form in which the theorem of [18] is implemented with the constraints for background fields of [7]. In section 4 we propose a one-loop anomaly for the BRST symmetry and show that it is cohomologically non trivial. Finally, we end with a conclusion section. Before finish this section, we shall introduce our notation.

1.1 Notation

Our normalization for n-(super)forms is

$$F = \frac{1}{n!} dZ^{M_1} \dots dZ^{M_n} F_{M_n \dots M_1} = \frac{1}{n!} E^{A_1} \dots E^{A_n} F_{A_n \dots A_1}, \tag{1.1}$$

where Z^M are the ten dimensional $N = 1$ superspace coordinates, $E^A = dZ^M E_M{}^A$ are the supervielbeins. We use latin letters for vector-like indices, greek letters for spinor-like

[1] A procedure to obtain corrections to SUGRA/SYM system order by order in α' was done in [21]. Unfortunately, this approach, as developed in [21], leads to inconsistencies (see [22]).

indices and Capital letters for both. Letters from the beginning of the alphabet denote flat (Lorentz) indices and letters from the middle of alphabet are for curved ones. Once a set of supervielbeins is specified, an n superform can be decomposed as

$$F = \sum F_{p,q}, \tag{1.2}$$

where $F_{p,q}$ denote the component of F with p vector-like vielbeins and $q = n - p$ spinor-like vielbeins.

2. The heterotic string action

The sigma model action for the heterotic string in a SUGRA/SYM background in the pure spinor formalism is given by [7]

$$\begin{aligned} S = \frac{1}{\alpha'} \int d^2z \Big[& \frac{1}{2}\Pi^a \overline{\Pi}^b \eta_{ab} + \frac{1}{2}\Pi^A \overline{\Pi}^B B_{BA} + \omega_\alpha \overline{\nabla} \lambda^\alpha \\ & + d_\alpha (\overline{\Pi}^\alpha + \overline{J}^I W_I^\alpha) + \Pi^A A_{IA} \overline{J}^I + \lambda^\alpha \omega_\beta \overline{J}^I U_{I\alpha}{}^\beta \Big] + S_J + S_{FT}, \end{aligned} \tag{2.1}$$

where $(\Pi^A, \overline{\Pi}^A) = (\partial Z^M E_M{}^A, \overline{\partial} Z^M E_M{}^A)$, λ^α is a pure spinor and ω_α is its conjugate momentum. The covariant derivative for the pure spinor λ^α is given by $\overline{\nabla}\lambda^\alpha = \overline{\partial}\lambda^\alpha + \lambda^\beta \overline{\partial} Z^M \Omega_{M\beta}{}^\alpha$, where $\Omega_{M\alpha}{}^\beta$ is the connection for the structure group and it has the form $\Omega_{M\alpha}{}^\beta = \Omega_M^{(s)} \delta_\alpha{}^\beta + \frac{1}{4}\Omega_{\mathrm{Mab}}(\gamma^{ab})_\alpha{}^\beta$. The world-sheet field d_α has conformal weight $(1,0)$ and plays the role of generating translations in superspace. $\overline{J}^I$ $(I = 1, \ldots, 496)$, with conformal weight $(0,1)$, are the currents of the gauge group, SO(32) or $E_8 \times E_8$ and $dZ^M A_{IM}$ is the gauge group connection. S_J is the free action for the heterotic fermions. The superfield W_I^α has the gaugino as the lowest component and $U_{I\alpha}{}^\beta$ contains the field strength for the gauge boson in its lowest component. Finally, S_{FT} is the Fradkin-Tseytlin term given by

$$S_{FT} = \int d^2z \; r^{(2)} \Phi, \tag{2.2}$$

where $r^{(2)}$ is the world-sheet curvature and Φ is the dilaton superfield. Although the Fradkin-Tseytlin term breaks the classical conformal invariance of the action (2.1), it helps to restore it at the quantum level as it was shown in [15] in the one-loop case. Note that the dilaton superfield is related to the Weyl part of the curvature connection as $\nabla_\alpha \Phi = 4\Omega_\alpha^{(s)}$.

Besides the action (2.1) being classically invariant under conformal transformations, it is invariant under gauge transformations and a pair of Lorentz transformations acting on the background fields. These two Lorentz transformations act independently on the bosonic local indices, e.g. $\delta \Pi^a = \Pi^b \Sigma_b{}^a$, and on the fermionic local indices, e.g. $\delta d_\alpha = -\Sigma_\alpha{}^\beta d_\beta$. Both Lorentz transformations can be identified as it is done in [7].

The pure spinor superstring has a very important symmetry, it is invariant under the BRST-like pure spinor transformation [1] generated by the pure spinor BRST charge $Q = \oint \lambda^\alpha d_\alpha$. As it was stressed in [7], one must demand that also the action (2.1) is invariant under this symmetry. By demanding nilpotence and world-sheet time conservation of Q, the action (2.1) turns out to be invariant if the background superfields satisfy suitable constraints which determine the SUGRA/SYM equations of motion for them. Nilpotence is achieved by demanding

$$\lambda^\alpha \lambda^\beta T_{\alpha\beta}{}^A = \lambda^\alpha \lambda^\beta H_{\alpha\beta A} = \lambda^\alpha \lambda^\beta F_{I\alpha\beta} = \lambda^\alpha \lambda^\beta \lambda^\gamma R_{\alpha\beta\gamma}{}^\delta = 0, \tag{2.3}$$

where T^A is the torsion 2-form, $H = dB$, F is the field-strength two form and $R_\alpha{}^\beta$ is the curvature two form. We use the notation of [15]. The charge conservation can be obtained by determining the equations of motion of (2.1) and then imposing $\overline{\partial}(\lambda^\alpha d_\alpha) = 0$ [7] or by demanding invariance of (2.1) under the BRST transformations [8]. In this case, the action transforms as

$$\begin{aligned} QS = \frac{1}{\alpha'}\int d^2z \Bigg[& \frac{1}{2}\lambda^\alpha \Pi^a \overline{\Pi}^b (T_{\alpha(ab)} + H_{ba\alpha}) + \frac{1}{2}\lambda^\alpha \Pi^\beta \overline{\Pi}^a (H_{\beta\alpha a} - T_{\beta\alpha a}) + \lambda^\alpha d_\beta \overline{\Pi}^a T_{a\alpha}{}^\beta \\ & - \lambda^\alpha \lambda^\beta d_\gamma \overline{\Pi}^a R_{a\alpha\beta}{}^\gamma + \lambda^\alpha \Pi^a \overline{J}^I \left(\frac{1}{2}(H_{\alpha\beta a} + T_{\alpha\beta a}) W_I^\beta - F_{Ia\alpha}\right) \\ & + \lambda^\alpha \Pi^\beta \overline{J}^I \left(\frac{1}{2} H_{\alpha\beta\gamma} W_I^\gamma - F_{I\alpha\beta}\right) + \lambda^\alpha d_\beta \overline{J}^I (U_{I\alpha}{}^\beta - W_I^\gamma T_{\gamma\alpha}{}^\beta - \nabla_\alpha W_I^\beta) \\ & + \lambda^\alpha \lambda^\beta \omega_\gamma (\nabla_\alpha U_{I\alpha}{}^\gamma + W_I^\delta R_{\delta\alpha\beta}{}^\gamma) \Bigg]. \end{aligned} \tag{2.4}$$

As it was shown in [7], the nilpotence constraints (2.3) and the vanishing of (2.4) allow to write the following constraints for the torsion and curvature components

$$T_{a\alpha}{}^\beta = T_{\alpha\beta}{}^\gamma = 0, \qquad T_{\alpha\beta}{}^a = \gamma^a_{\alpha\beta}, \qquad T_{\alpha a}{}^b = 2(\gamma_a{}^b)_\alpha{}^\beta \Omega_\beta, \tag{2.5}$$

$$H_{\alpha\beta\gamma} = H_{a\beta\gamma} - (\gamma_a)_{\beta\gamma} = 0, \tag{2.6}$$

$$F_{I\alpha\beta} = 0, \tag{2.7}$$

where $\gamma^a_{\alpha\beta}$ and $(\gamma^a)^{\alpha\beta}$ denote the usual Pauli matrices, i.e. the off-diagonal blocks of the Dirac matrices, so that they are symmetric in (α, β). Besides, Bianchi identities imply that the torsion component $T_{abc} = \eta_{cd} T_{ab}{}^d$ is completely antisymmetric [15]. Note that the torsion component $T_{\alpha\beta}{}^\gamma$ can be set to zero only after the use of the 'shift' symmetry of [7].

Note that in (2.4), the field equation

$$\overline{\Pi}^\alpha + \overline{J}^I W_I^\alpha = 0, \tag{2.8}$$

which follows from varying the action (2.1) respect to the world-sheet field d_α, has been used.

Finally one must require that the action (2.1) is also invariant under the "ω-symmetry" $\delta\omega_\alpha = (\gamma^a \lambda)_\alpha \Lambda_a$, where Λ_a are local parameters, which implies that

$$U_{I\alpha}{}^\beta = U_I \delta_\alpha{}^\beta + U_{Iab}(\gamma^{ab})_\alpha{}^\beta.$$

A natural question to be addressed at this point is the quantum preservation of the symmetries of (2.1). In particular, the possibility of finding α' corrections to the constraints of (2.3) and those from the vanishing of (2.4). Let us first discuss the gauge and Lorentz anomalies.[2]

The anomaly for the local symmetries is

$$\delta\Gamma_{\rm eff} = \int d^2z \left[\frac{1}{2}(\overline{\partial}A_I - \partial\overline{A}_I)\varepsilon_I + \frac{1}{4}(\overline{\partial}\Omega_{ab} - \partial\overline{\Omega}_{ab})\Sigma^{ab}\right], \tag{2.9}$$

where $\Gamma_{\rm eff}$ is the effective action (i.e. the generating functional of 1PI vertex functions), ε_I and Σ^{ab} are the parameters of the gauge and the Lorentz transformations respectively and $A_I = \partial Z^M A_{IM}, \overline{A}_I = \overline{\partial} Z^M A_{IM}, \Omega_{ab} = \partial Z^M \Omega_{\rm Mab}, \overline{\Omega}_{ab} = \overline{\partial} Z^M \Omega_{\rm Mab}$. It is also possible the presence of terms like

$$\int d^2z\ d_\alpha W_I^\alpha \overline{\partial}\varepsilon_I,$$

which can be eliminated by adding suitable counterterms. There is also a potential anomaly associated to $\Omega^{(s)}$. However, this contribution vanishes because $\Omega^{(s)}$ appears in the combination $\Omega^{(s)} + \overline{J}^I U_I$ which is zero on-shell [15].

Since the quantum theory cannot be anomalous under a local symmetry, the expression (2.9) must be canceled by the standard Green-Schwarz mechanism [17]. It is done by allowing the B two-form superfield not to be inert under gauge and Lorentz transformations. It has to transform as

$$\delta B = -\alpha'\left(dA_I\varepsilon_I + \frac{1}{2}d\Omega_{ab}\Sigma^{ab}\right). \tag{2.10}$$

In order to assure gauge and Lorentz invariance of the B field strength H one must define it as

$$H = dB - \frac{\alpha'}{2}\omega^{(CS)}, \tag{2.11}$$

where $\omega^{(CS)}$ is the Chern-Simons three from given by

$$\omega^{(CS)} = tr(AdA - \frac{2}{3}A^3) + \Omega^{ab}d\Omega_{ab} - \frac{2}{3}\Omega_a{}^b\Omega_b{}^c\Omega_c{}^a, \tag{2.12}$$

and satisfying

$$dH = tr(FF) + R^{ab}R_{ab}. \tag{2.13}$$

Note that H in (2.11) is defined up to a gauge and Lorentz invariant three-form.

The classical constraints coming from (2.3) and the vanishing of (2.4) lead to $dH = 0$ and therefore have to be corrected. These corrections arise as anomalies of the BRST symmetry generated by the nilpotent charge Q, that is, if we define

$$Q\Gamma_{\rm eff} = \alpha'\mathcal{A}, \tag{2.14}$$

$\mathcal{A}$ is a non trivial cocycle of the cohomology of Q, in the space of local functionals of ghost number 1. Then from the previous discussion it is expected that $\mathcal{A}$ will contain a term

$$\mathcal{A} = \frac{1}{2}\int d^2z\ \lambda^\alpha \Pi^A \overline{\Pi}^B \omega^{(CS)}_{BA\alpha} + \cdots, \tag{2.15}$$

[2]The following paragraph is based on discussions with N. Berkovits and V. Pershin.

which modifies the definition of H since the variation of the term involving B in the action (2.1) is proportional to

$$\int d^2z\ \lambda^\alpha \Pi^A \overline{\Pi}^B (dB)_{BA\alpha}.$$

In the next sections we will determine the complete form of (2.15) by studying the conditions coming from $Q\mathcal{A} = 0$.

3. The cohomology and an useful theorem

Let us start from (2.15) and compute its BRST variation. Any variation of the Chern-Simons term showed in (2.15) is

$$\delta \int d^2z\ \lambda^\alpha \Pi^A \overline{\Pi}^B \omega^{(CS)}_{BA\alpha} = \int d^2z\ \delta Z^M E_M{}^C \lambda^\beta \Pi^A \overline{\Pi}^B (d\omega^{(CS)})_{BA\beta C},$$

because $\omega^{(CS)}$ is a three-form. Now we recall the BRST variation for Z^M to be [8]

$$\delta_{BRST} Z^M = Q Z^M = \lambda^\alpha E_\alpha{}^M,$$

then we obtain

$$\begin{aligned} Q\mathcal{A} &= \int d^2z\ \frac{1}{2}\lambda^\alpha \lambda^\beta \Pi^A \overline{\Pi}^B (d\omega^{(CS)})_{BA\alpha\beta} + \cdots \\ &= \int d^2z\ \lambda^\alpha \lambda^\beta \Pi^A \overline{\Pi}^B \left(F_I F_I + \frac{1}{2} R^{ab} R_{ab} \right)_{BA\alpha\beta} + \cdots, \end{aligned} \tag{3.1}$$

where we have used $d\omega^{(CS)} = tr(FF) + R^{ab}R_{ab}$. We will fix the $\cdots$ terms to make this expression to vanish.

It follows from the constraint $F_{I\alpha\beta} = 0$, that the 4-superform $F_I F_I$ vanishes in the sectors $(0,4)$ and $(1,3)$ (i.e. in the sectors with 4 or 3 spinor-like local indices). Moreover in the sector $(2,2)$ $(F_I F_I)_{ba\alpha\beta}$ has the following structure

$$(F_I F_I)_{ba\alpha\beta} = (\gamma_{[a})_{\alpha\gamma} (\gamma_{b]})_{\beta\delta} W_I^\gamma W_I^\delta. \tag{3.2}$$

As it will be shown in section 4, this structure is essential to compute the anomaly for the gauge part in (3.1) . The curvature part in (3.1) could be treated similarly if the index structure were the same. Unfortunately, it is not the case with the constraints of (2.3) and (2.4). However there exists the following result. It was shown in [13, 14, 19] that with a different set of torsion constraints [18] (the gauge part has the same constraints) that

$$R'^{ab} R'_{ab} = dX' + K, \tag{3.3}$$

where the three form X' and the four form K are Lorentz invariant. They were determined in [19]. The main property in (3.3) is that the four form K vanishes in the sectors $(0,4)$ and $(1,3)$ and that in the sector $(2,2)$ has the same structure than $(F_I F_I)_{ab\alpha\beta}$, that is

$$K_{ab\alpha\beta} = (\gamma_{[a})_{\alpha\gamma} (\gamma_{b]})_{\beta\delta} K^{\gamma\delta},$$

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with $K^{\gamma\delta} = -K^{\delta\gamma}$. This property will be crucial to determine also the Lorentz part in the BRST anomaly.

In order to use this result we should relate the Berkovits-Howe constraints to the ones of [19]. Now it will be shown that there exists a redefinition of fields which makes the job. We redefine the vielbein one-form as

$$E'^a = e^{-\frac{1}{3}\Phi}E^a, \quad E'^\alpha = e^{-\frac{1}{6}\Phi}\left(E^\alpha + \frac{1}{3}E^a\gamma_a^{\alpha\beta}\nabla_\beta\Phi\right), \tag{3.4}$$

and the components of the Lorentz superspace connection one form as

$$\Omega'_{ab} = \Omega_{ab} + \Lambda_{ab}, \tag{3.5}$$

where

$$\Lambda_{ab} = \frac{1}{3}E_{[a}\nabla_{b]}\Phi - \frac{1}{12}E^c\gamma_{cab}^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi - \frac{1}{6}E^\alpha(\gamma_{ab})_\alpha{}^\beta\nabla_\beta\Phi. \tag{3.6}$$

In components, these transformations imply the following torsion constraints

$$T'_{\alpha\beta}{}^a = \gamma_{\alpha\beta}^a, \quad T'_{\alpha\beta}{}^\gamma = T'_{\alpha a}{}^b = 0, \quad T'_{a\alpha}{}^\beta = \frac{1}{3}e^{\frac{1}{6}\Phi}(\gamma_a\gamma^{bcd})_\alpha{}^\beta\tau_{bcd}, \tag{3.7}$$

where

$$\tau_{bcd} = \frac{1}{96}\gamma_{bcd}^{\gamma\delta}\left(\nabla_\gamma\nabla_\delta\Phi + \frac{4}{3}(\nabla_\gamma\Phi)(\nabla_\delta\Phi)\right),$$

which correspond to a set of constraints used in [19] to show the theorem (3.3).

Now we can use (3.5) to rewrite (3.3) for the Berkovits-Howe constraints. In fact,

$$R^{ab}R_{ab} = R'^{ab}R'_{ab} - d(2R_{ab}\Lambda^{ab} + \Lambda_{ab}\nabla\Lambda^{ab}), \tag{3.8}$$

Therefore, we have shown that

$$R^{ab}R_{ab} = dX + K, \tag{3.9}$$

where

$$X = X' - 2R_{ab}\Lambda^{ab} - \Lambda_{ab}\nabla\Lambda^{ab}.$$

4. Quantum BRST invariance

From the discussion of the previous section it is expected that the anomaly is

$$\mathcal{A} = \frac{1}{2}\int d^2z\ \lambda^\alpha\Pi^A\overline{\Pi}^B\widehat{\omega}_{BA\alpha} + \cdots, \tag{4.1}$$

where

$$\widehat{\omega} = \omega^{(CS)} - X, \tag{4.2}$$

and

$$d\widehat{\omega} = L,$$

where the closed 4-superform L is

$$L = 2F_IF_I + R_{ab}R^{ab} - dX = 2F_IF_I + K. \tag{4.3}$$

Note that L vanishes in the sectors $(0,4)$ and $(1,3)$ and in the sector $(2,2)$ its flat components have the following structure

$$L_{ba\alpha\beta} = (\gamma_{[a})_{\alpha\gamma}(\gamma_{b]})_{\beta\delta}L^{\gamma\delta},$$

where

$$L^{\alpha\beta} = K^{\alpha\beta} + W_I^\alpha W_I^\beta. \tag{4.4}$$

Moreover $L^{\gamma\delta}$ belongs to the 120 representation of the Lorentz group and therefore can be written as

$$L^{\beta\gamma} = \gamma_{abc}^{\beta\gamma}L^{abc}. \tag{4.5}$$

The BRST variation of $\mathcal{A}$ will be of the form

$$Q\mathcal{A} = \frac{1}{2}\int d^2z\ \lambda^\alpha\lambda^\beta\Pi^A\overline{\Pi}^B L_{BA\alpha\beta} + \cdots. \tag{4.6}$$

In the case of the GS heterotic string the analogous of (4.1) (without $\cdots$) is the all story. Indeed in this case, the anomaly and its variation are still given by (4.1) and (4.6) but with λ^α replaced by $\delta\kappa_\gamma\Pi^c\gamma_c^{\gamma\alpha}$. With this substitution (4.6) (without $\cdots$) vanishes (modulo the Virasoro constraint) and (4.1) (without $\cdots$) is the full consistency anomaly.

For the pure spinor string one must supplement the first term in the r.h.s. of (4.1) with further terms (represented by the $\cdots$) in order to recover a consistent anomaly. We shall show that the pure spinor BRST anomaly is

$$\mathcal{A} = \frac{1}{2}\int d^2z\ [\lambda^\alpha\Pi^A\overline{\Pi}^B\widehat{\omega}_{BA\alpha} - \lambda^\alpha d_\beta\overline{\Pi}^a(\gamma_a)_{\alpha\gamma}L^{\beta\gamma} - \lambda^\alpha\lambda^\beta\omega_\gamma\overline{\Pi}^a(\gamma_a)_{\beta\rho}\nabla_\alpha L^{\gamma\rho}], \tag{4.7}$$

For that we must compute the BRST variation of $\mathcal{A}$ and show that $Q\mathcal{A}$ vanishes. The relevant BRST transformations are [8]

$$\begin{aligned} Q\Pi^A &= \delta_\alpha^A\nabla\lambda^\alpha - \lambda^\alpha\Pi^B T_{B\alpha}{}^A, \quad Q\lambda^\alpha = 0, \\ Qd_\alpha &= \lambda^\beta\Pi^a(\gamma_a)_{\beta\alpha} + \lambda^\beta\lambda^\gamma\omega_\delta R_{\alpha\beta\gamma}{}^\delta, \end{aligned} \tag{4.8}$$

and [1, 23]

$$Q\omega_\alpha = d_\gamma(\delta_\alpha^\gamma - \mathcal{K}^\gamma{}_\alpha), \tag{4.9}$$

where

$$\mathcal{K}^\gamma{}_\alpha = \frac{1}{2}(\gamma^a Y)^\gamma(\lambda\gamma_a)_\alpha$$

and $Y_\alpha = \frac{v_\alpha}{(v\lambda)}$ so that $(Y\lambda) = 1$, v_α being a constant spinor . Note that although we have added a non covariant object, namely $\mathcal{K}$, the final result is covariant. Now it will be shown that the anomaly (2.15) is invariant under the symmetry transformation $\delta\omega_\alpha = (\gamma_a\lambda)_\alpha\Lambda^a$. Then, the term $d_\gamma\mathcal{K}^\gamma{}_\alpha$ in (4.9) does not contributes and (4.9) can be replaced by $Q\omega_\alpha = d_\alpha$. To prove this consider first the gauge part in (2.15). After using $\nabla_\alpha W_I^\beta = U_{I\alpha}{}^\gamma$, we obtain

$$\nabla_\alpha(W_I^\beta W_I^\gamma) = U_I\delta_\alpha^{[\beta}W_I^{\gamma]} + \frac{1}{4}U_I{}^{ab}(\gamma_{ab})_\alpha{}^{[\beta}W_I^{\gamma]},$$

then plugging this into the variation under $\delta\omega_\alpha = (\gamma_a\lambda)_\alpha\Lambda^a$ we find that $\mathcal{A}$ varies as the integral of

$$\Lambda^b\overline{\Pi}^a\lambda^\alpha\lambda^\beta\lambda^\gamma(\gamma_b)_{\gamma\sigma}(\gamma_a)_{\beta\rho}\left[U_I\delta_\alpha^{[\sigma}W_I^{\rho]}+\frac{1}{4}(\gamma_{cd})_\alpha{}^{[\sigma}W_I^{\rho]}U_I^{cd}\right]$$
$$=\Lambda^b\overline{\Pi}^a\lambda^\alpha\lambda^\beta\lambda^\gamma\left[(\gamma_{[a})_{\alpha\beta}(\gamma_{b]})_{\gamma\rho}U_IW_I^\rho-\frac{1}{4}(\gamma_{cd}\gamma_{[a})_{\alpha\beta}(\gamma_{b]})_{\gamma\rho}U_I^{cd}W_I^\rho\right],$$

which vanishes because the pure spinor condition.

Similarly, for the K-part we need to use the result [19] and the mappings (3.4), (3.5). We first define $K^{\beta\gamma}=\gamma_{abc}^{\beta\gamma}K^{abc}$ to obtain

$$\nabla_\alpha K^{\beta\gamma}=(\gamma^{abc})^{\beta\gamma}\nabla'_\alpha K_{abc}+2\Omega_\alpha^{(s)}K^{\beta\gamma}-(\gamma^{abc})^{\beta\gamma}(\gamma_a{}^d)_\alpha{}^\rho K_{bcd}\Omega_\rho^{(s)},$$

where

$$\nabla'_\alpha K_{abc}=(\gamma_{[a})_{\alpha\beta}K_{bc]}{}^\beta,$$

as it was shown in [19]. Plugging this into the variation of (2.15) under the pure spinor gauge transformation, we obtain that the variation of (2.15) becomes the integral of

$$\Lambda^b\overline{\Pi}^a\lambda^\alpha\lambda^\beta\lambda^\gamma(\gamma_{abcde})_{(\alpha\beta}\gamma^c_{\gamma)\delta}(K^{de\delta}+\gamma_f^{\delta\rho}K^{def}\Omega_\rho^{(s)}),$$

which vanishes because of the identity

$$(\gamma_{abcde})_{(\alpha\beta}\gamma^c_{\gamma)\rho}=-\frac{1}{2}\gamma^c_{(\alpha\beta}(\gamma_{abde}\gamma_c)_{\gamma)\rho},$$

and the pure spinor constraint.

Now let us compute $Q\mathcal{A}$. It is not difficult to obtain, after using (4.6), that

$$Q\mathcal{A}=\frac{1}{2}\int d^2z\,\Big(\lambda^\alpha\lambda^\beta d_\gamma\overline{\Pi}^a[-T_{a\alpha}{}^\beta(\gamma_b)_{\beta\rho}L^{\gamma\rho}+\nabla_\alpha((\gamma_a)_{\beta\rho}L^{\gamma\rho})-(\gamma_a)_{\alpha\rho}\nabla_\alpha L^{\gamma\rho}] \tag{4.10}$$
$$+\lambda^\alpha\lambda^\beta\lambda^\gamma\omega_\delta\overline{\Pi}^a[T_{a\alpha}{}^b(\gamma_b)_{\beta\rho}\nabla_\gamma L^{\delta\rho}-\nabla_\alpha((\gamma_a)_{\beta\rho}\nabla_\gamma L^{\delta\rho})+R_{\rho\alpha\beta}{}^\delta(\gamma_a)_{\gamma\sigma}L^{\rho\sigma}]\Big).$$

If note that $\nabla_\alpha(\gamma_a)_{\beta\gamma}=-2\Omega_\alpha^{(s)}(\gamma_a)_{\beta\gamma}$, the Fierz identity for the gamma matrices and the pure spinor condition, then the first line in (4.10) vanishes and we are left with the expression from the last line that contains

$$\lambda^\alpha\lambda^\beta\lambda^\gamma[R_{\rho\alpha\beta}{}^\delta L^{\rho\sigma}+\nabla_\alpha\nabla_\beta L^{\delta\sigma}](\gamma_a)_{\gamma\sigma}. \tag{4.11}$$

If we symmetrize in $(\alpha\beta)$, use

$$\{\nabla_\alpha,\nabla_\beta\}L^{\delta\sigma}=-T_{\alpha\beta}{}^A\nabla_AL^{\delta\sigma}+L^{\rho\sigma}R_{\alpha\beta\rho}{}^\delta+L^{\delta\rho}R_{\alpha\beta\rho}{}^\sigma,$$

and the Bianchi identity $R_{(\alpha\beta\rho)}{}^\delta=0$, then we obtain that (4.11) is proportional to

$$\lambda^\alpha\lambda^\beta\lambda^\gamma R_{\alpha\beta\rho}{}^\sigma(\gamma_a)_{\gamma\sigma}. \tag{4.12}$$

But

$$R_{\alpha\beta\rho}{}^\sigma=\delta_\rho^\sigma R_{\alpha\beta}+\frac{1}{4}(\gamma^{bc})_\rho{}^\sigma R_{\alpha\beta bc}.$$

If we plug this expression into (4.12), we see that $R_{\alpha\beta}$ does not contribute because it contains a term like $\gamma^c_{\alpha\beta}$. Analogously $R_{\alpha\beta bc}$ is expressed in terms of a term along $\gamma^d_{\alpha\beta}$, which again does not contribute, and a term along γ_{bcdef}. We note that this contribution also vanishes because of the identity

$$(\gamma_a\gamma^{bc})_{\rho(\alpha}(\gamma_{bcdef})_{\beta\gamma)} = (\gamma_a\gamma^b)_\rho{}^\sigma\gamma^c_{\sigma(\alpha}(\gamma_{bcdef})_{\beta\gamma)} = \frac{1}{2}(\gamma_a\gamma^b)_\rho{}^\sigma(\gamma_c\gamma_{bdef})_{\sigma(\alpha}\gamma^c_{\beta\gamma)}.$$

Therefore we have obtained

$$Q\mathcal{A} = 0. \tag{4.13}$$

The anomaly $\mathcal{A}$ in (2.15) can be absorbed by relaxing the torsion and curvature constraints that follow from (2.3) and the vanishing of (2.4) and by modifying them. In fact we can impose

$$\begin{aligned}
QS - \alpha'\mathcal{A} = \frac{1}{\alpha'}\int d^2z\Big[&\frac{1}{2}\lambda^\alpha\Pi^a\overline{\Pi}^b(T_{\alpha(ab)} + \widehat{H}_{ba\alpha}) + \frac{1}{2}\lambda^\alpha\Pi^\beta\overline{\Pi}^a(\widehat{H}_{\beta\alpha a} - T_{\beta\alpha a}) \\
&+\lambda^\alpha\Pi^a\overline{J}^I\left(\frac{1}{2}(\widehat{H}_{\alpha\beta a} + T_{\alpha\beta a})W^\beta_I - F_{Ia\alpha}\right) - \lambda^\alpha\Pi^\beta\overline{J}^I(W^\gamma_I\widehat{H}_{\gamma\alpha\beta} + F_{I\alpha\beta}) \\
&+\lambda^\alpha d_\beta\overline{J}^I(U_{I\alpha}{}^\beta - W^\gamma_I T_{\gamma\alpha}{}^\beta - \nabla_\alpha W^\beta_I) + \lambda^\alpha\lambda^\beta\omega_\gamma(\nabla_\alpha U_{I\alpha}{}^\gamma + W^\delta_I R_{\delta\alpha\beta}{}^\gamma) \\
&+\lambda^\alpha d_\beta\overline{\Pi}^a\left(T_{a\alpha}{}^\beta - \frac{\alpha'}{2}(\gamma_a)_{\alpha\gamma}L^{\beta\gamma}\right) - \lambda^\alpha\lambda^\beta\overline{\Pi}^a\left(R_{a\alpha\beta}{}^\gamma - \frac{\alpha'}{2}(\gamma_a)_{\delta(\alpha}\nabla_{\beta)}L^{\gamma\delta}\right)\Big] = 0,
\end{aligned} \tag{4.14}$$

where we have defined

$$\widehat{H} = dB - \frac{\alpha'}{2}\widehat{\omega}. \tag{4.15}$$

Equation (4.14) means that the structure of the anomaly is such that a violation of the BRST invariance of the classical action S, represented by a change of the constraints, can be chosen so that it cancels the anomaly, as in the GS mechanism.

It follows from (4.14) that the constraints

$$T_{\alpha\beta}{}^\gamma = 0, \quad T_{\alpha\beta}{}^a = \gamma^a_{\alpha\beta}, \quad T_{\alpha a}{}^b = 2(\gamma_a{}^b)_\alpha{}^\beta\Omega_\beta \quad F_{I\alpha\beta} = 0,$$

remain the same. Only the constraints (2.6) are changed in the sense that it is $\widehat{H}$ and not H that satisfies these constraints. All the other components of the torsion and curvatures follow from the Bianchi identities. In particular

$$T_{a\alpha}{}^\beta = \frac{\alpha'}{2}(\gamma_a)_{\alpha\gamma}L^{\beta\gamma},$$

and

$$\lambda^\alpha\lambda^\beta R_{a\alpha\beta}{}^\gamma = \alpha'\lambda^\alpha\lambda^\beta(\gamma_a)_{\delta\alpha}\nabla_\beta L^{\gamma\delta},$$

in agreement of the last two terms of (4.14).

We expect that the corrections we have found will induce a correction in the nilpotence of the BRST charge, at the one-loop level, that consists in replacing H with $\widehat{H}$ in the second constraint in (2.3).

5. Concluding remarks

In this paper we have obtained the corrections of order α' for the constraints of the σ-model of the pure spinor heterotic string, that implement the GS anomaly cancelation mechanism. They arise as anomalies of the BRST charge. In fact, having worked at a cohomological level, we have obtained the general form of these corrections, which depends on two unspecified constants: one in front of the gauge anomaly and one in front of the Lorentz one. These constants are fixed as in (2.15) by requiring that the variations of B under gauge and Lorentz transformations, induced by this BRST anomaly, cancel the gauge and Lorentz anomalies (2.9), according to the GS mechanism. It could be interesting to check these values of the constants by an explicit one loop calculation.

We have obtained our result in the framework of the set of constraints found in [7] starting from (2.1). A redefinition of the supervielbeins and superconnections leads to a different but equivalent set of constraints. Of course the redefinition changes the σ-model action (2.1) but the new action is equally suited and gives rise to equivalent results. For instance the redefinitions (3.4) and (3.5) lead to the action

$$
\begin{aligned}
S = \frac{1}{\alpha'}\int d^2z\ \Bigg[&\frac{1}{2}e^{\frac{2}{3}\Phi}\Pi^a\overline{\Pi}^b\eta_{ab} + \frac{1}{2}\Pi^A\overline{\Pi}^B B_{BA} + \Pi^A A_{IA}\overline{J}^I + \omega_\alpha\overline{\nabla}\lambda^\alpha \\
&+d_\alpha\left(\overline{\Pi}^\alpha + \overline{J}^I W_I^\alpha - \frac{1}{3}\overline{\Pi}^a\gamma_a^{\alpha\beta}\nabla_\beta\Phi\right) + \lambda^\alpha\omega_\beta\left(\overline{J}^I U_{I\alpha}{}^\beta + \frac{1}{4}\overline{\Pi}^A\Lambda_A{}^{ab}(\gamma_{ab})_\alpha{}^\beta\right)\Bigg] \\
+S_J + S_{FT},&
\end{aligned}
$$

where Λ^{ab} is the one form (3.6) expressed in terms of E'^A (in this equation the suffix " ' " is suppressed).

Notice that a change in the action S not only changes the constraints coming from the vanishing of (2.4) but also induces changes in the definition of d_α and therefore gives rise to possible changes in the nilpotence motivated constraints (2.3). Also notice that the anomaly $\mathcal{A}$ is defined modulo a trivial cocycle that amounts to a modification of the action corresponding to an (α'-dependent) redefinition of supervielbeins, B-superform and superconnections [24].

In [25, 24], (see also [26]) an interesting set of constraints is proposed. For this set, the curvature R^{ab} in the sector $(0,2)$ vanishes at the classical level (zero order in α') and the 3-superform X is of order α' so that it does not contribute in $\widehat{H}$ at first order in α'. Then (4.15) looks as $\widehat{H} = dB - \frac{\alpha'}{2}\omega^{(CS)} - \alpha'^2 X$.

As it was shown in [19], the explicit solution of the Bianchi identities in the presence of the superform X leads to an unexpected and, at first sight, unpleasant feature: the solution contains poles that represent spurious states of negative norm (poltergheists) at a mass of the order $\frac{\kappa}{\alpha'}$ where κ is the v.e.v. of the dilaton. The poltergheists are the signal of a conflict between our requirements of supersymmetry, locality and unitarity (absence of anomalies). However one should not worry of them. Indeed the spurious poles arise at a very high mass in a region of energy where our perturbative expansion in α' is clearly unreliable. This is similar, after all, to what happens in the well-known low energy effective actions of gravity with terms quadratic in the curvature, which also contain poltergheists.

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Notice that in the set of constraints of [25, 24], X does not contribute at first order in α' and therefore the spurious poles appear only at higher orders. Moreover the spurious states can be decoupled at any finite order in α', at the expense of locality, by solving recursively the relevant equations, as discussed in [19].

As a last remark, let us notice that the cohomological method of this paper could be used to search for anomalies and corrections of the constraints, for type II superstrings and/or for heterotic strings at higher order in α'. In particular it should be interesting to search for the anomaly at the order α'^3 that corresponds to the celebrated "R^4" term in the action and would provide for the supersymmetrization of this term. Previous attempts in this direction (for the heterotic GS string) are in [27]. Note that the complete R^4 terms for the type II superstrings were obtained recently in [28] by using tree-level scattering amplitudes in the pure spinor formalism.

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